

How do I reduce this Matrix!

$$A = \begin{bmatrix} 1 & 2 & 4 \\ \textcircled{4} & 7 & 10 \\ -2 & 1 & 8 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

We would like this to be "0". First we multiply the top row (R_1) by -4.

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & 7 & 10 \\ -2 & 1 & 8 \end{bmatrix} \xrightarrow{-4R_1} \begin{bmatrix} -4 & -8 & -16 \\ 4 & 7 & 10 \\ -2 & 1 & 8 \end{bmatrix}$$

Now, we add R_1 to R_2 , the top row to the middle row

$$\begin{bmatrix} -4 & -8 & -16 \\ 4 & 7 & 10 \\ -2 & 1 & 8 \end{bmatrix} \xrightarrow{R_1 + R_2} \begin{bmatrix} -4 & -8 & -16 \\ 0 & -1 & -6 \\ -2 & 1 & 8 \end{bmatrix}$$

Now, we want to reduce this -2 to 0. We use the same method, except we add R_1 to R_3 this time, and we have to ~~add~~ multiply R_1 by $\frac{1}{2}$.

* Note: We will reduce this to 1 step *

$$\begin{bmatrix} -4 & -8 & -16 \\ 0 & -1 & -6 \\ -2 & 1 & 8 \end{bmatrix} \xrightarrow{\left(\frac{1}{2}R_1\right) + R_3} \begin{bmatrix} 2 & 4 & 8 \\ 0 & -1 & -6 \\ 0 & \textcircled{5} & 16 \end{bmatrix}$$

Now we have this matrix, $\begin{bmatrix} 2 & 4 & 8 \\ 0 & -1 & -6 \\ \textcircled{0} & \textcircled{5} & 16 \end{bmatrix}$

We want to reduce this $\textcircled{5}$ to a zero and keep the $\textcircled{0}$ zero as well. The easiest way to do this is to multiply R_2 by 5 and add it to R_3 .

* Note: We will make this a single step again

$$\begin{bmatrix} 2 & 4 & 8 \\ 0 & -1 & -6 \\ 0 & 5 & 16 \end{bmatrix} \xrightarrow{(5R_2) + R_3} \begin{bmatrix} \textcircled{2} & 4 & 8 \\ 0 & \textcircled{-5} & -30 \\ 0 & 0 & -14 \end{bmatrix}$$

Almost done, we now want to reduce the 2 to a 1 and the -5 to a 1 (and the -14 to a 1). We use multiplication to do this.

$$\begin{bmatrix} 2 & 4 & 8 \\ 0 & -5 & -30 \\ 0 & 0 & -14 \end{bmatrix} \xrightarrow{(\frac{1}{2})R_1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -30 \\ 0 & 0 & -14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -30 \\ 0 & 0 & -14 \end{bmatrix} \xrightarrow{(\frac{-1}{5})R_2} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & -14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & -14 \end{bmatrix} \xrightarrow{(\frac{-1}{14})R_3} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

and finally,

This is the Reduced Echelon Form of our original matrix.

$$\text{ref}(A) = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$